Abstract: This paper is a continuation of our study of the effective acoustic behavior of porous media with microstructure. While in our previous works we considered the monophasic case (for both periodic and random geometry) [6, 7, 8, 9], here we focus on the situation when fluid and solid move out of phase. This is the case when the ratio of elasticity coefficients to viscosity coefficients is of order $\varepsilon^2$ and it corresponds to the dilatational wave of the second kind or the fast wave predicted by Biot theory [1]. We study the problem of derivation of an effective model of acoustic wave propagation in a two-phase, non-periodic medium modeling a fine mixture of linear elastic solid and a viscous Newtonian fluid. Bone tissue is an important example of a composite material that can be modeled in this fashion. We extend known homogenization results for periodic geometries to the case a stationary random, scale-separated microstructure. The ratio $\varepsilon$ of the macroscopic length scale and a typical size of the microstructural inhomogeneity is a small parameter of the problem. We employ stochastic two-scale convergence in the mean to pass to the limit $\varepsilon \to 0$ in the governing equations. The effective model is a biphasic phase viscoelastic material with long time history dependence. In this paper, as in our recent work [8], we assume that an elastic medium is randomly fissured with the associated random field being statistically homogeneous, with built-in scale separation. The effective equations are derived using the stochastic two-scale convergence in the mean [4, 10, 3]. Elimination of the corrector displacements is explained in detail for the periodic geometry. Considering periodic geometry is not a restriction since the periodic approximation procedure in the limit produces the same effective properties as the stochastic homogenization as showed by Bourgeat and Piatnitski [2] (see also Byström et al. [5] for some numerical examples). The effective equations model a biphasic macroscopic behavior with long-time history dependence.

This is a joint work with Alexander Panchenko (Washington State University) and Ana Vasilic (United Arab Emirates University).

References


